Research on Risk Hedging with Stock Index Futures in China

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Abstract The launch of HuShen 300 index futures contracts, which can improve the structure of China's stock market, will provide a way for the domestic stock market to hedge risk. On the basis of the introduction of the best HuShen 300 index futures hedging strategy, this paper focuses on the calculation of the optimal HuShen 300 index futures hedging ratio. Being different from most of the studies, this paper proposes a composite hedging ratio calculation approach and derives D-BEKK-ECM-BGARCH hedging ratio calculation model from the futures pricing theory. Empirical evidence on the hedging effect of the two models suggests that the stock portfolio's risk of yield can be significantly reduced after hedging and the application of the D-BEKK-ECM-BGARCH model can get the minimum transaction costs without losing hedging effect.
Key words HuShen 300 index futures; Optimal hedging ratio; CAPM model; ECM-GARCH model

1 Introduction

HuShen 300 index which is jointly issued by Shanghai and Shenzhen stock exchanges can reflect the overall trend A-share market. The index will play an important role in the development process of China's securities market. As the result of continuous improvement of China's market index system and innovation of financial markets, the launch of its futures contracts will be bound to be used widely as a tool to hedge financial risk by market participants .Till now, the research of the best hedging ratio of domestic stock index futures is still in its infancy, few scholars carried out detailed study on index futures optimal hedging ratio, this is mainly due to the lack of formal stock index futures contracts which can reflect the overall trend of Chinese A-share market .Most foreign scholars(Juan A.Lafuente,Alfonso Novales[1], Darren Butterworth, Phil Holmes[2], Figlewski S[3], Chuw[4], Park, T·H·and L·N·Switzer[5], Donald Lien Yiu Kuen Tse and Albert Tsui[6])use the European and American matured stock index futures market's data to calculate the hedging ratio with a variety of optimal hedging model and compare the efficiency of those hedging ratio empirically to choose the best one. Because China has not yet launched HuShen 300 stock index futures contracts formally, this article will take advantage of HuShen 300 index futures simulated trading market to study the best hedge ratio .Being different from other related research，this article will base on the futures pricing theory to calculate the optimal hedge ratio, and we will use the HuShen 300 index futures simulated trading market's date to study the best hedge ratio .

2 Related Theories and Models

2.1 Basic ideas

Because the underlying asset of index futures contracts is not a stock itself, but a stock market price index, in the process of calculating the hedge ratios, we should consider the relationship between stock and the spot stock index as well as the relationship between the spot stock index and stock index futures. As a result，we often use a complex method to calculate the hedging ratio of a single stock or a stock portfolio：Firstly, we calculate the stock's β risk factor with CAPM model, secondly, we calculate the stock index's hedging ratio h according to the principle of minimum variance. Then, the optimal hedge ratio is $\beta * h$.

2.2 β Risk factor

In CAPM model：

$$E(Rp)=R_f+\beta[E(R_M)-R_f] \tag{1}$$

Where $E(R_P)$ is the expected yield of a stock portfolio， R_f is the risk-free interest rate， $E(R_M)$ is the expected yield of the market portfolio，β is the system risk factor of a stock portfolio. We use regression analysis method to calculate β of a stock portfolio in this article.

2.3 The hedge ratio h

●Firstly, assume that price changes in spot stock index obey the geometric Brownian motion:

$$dS_t = u_{s,t} S_t dt + \sigma_{s,t} S_t dz_{1,t} \tag{2}$$

Where S_t is spot stock index, $u_{s,t}$ is the conditional mean of the yield, $\sigma_{s,t}$ is the conditional variance of the yield, $dz_{1,t} = \varepsilon_{1t}\sqrt{t}$, ε_{1t} obey the standard normal distribution independently.

●Secondly, assume that price changes in stock index futures obey the geometric Brownian motion:

$$dF_{t,T} = u_{f,t} F_{t,T} dt + \sigma_{s,t} F_{t,T} dz_{1,t} + \sigma_{N,t} F_{t,T} dz_{2,t} \tag{3}$$

Where $dz_{2,t} = \varepsilon_{2t}\sqrt{t}$, ε_{2t} obey the standard normal distribution independently. Assume that the Conditional correlation coefficient between ε_{1t} and ε_{2t} satisfy $\rho_{12,t} \neq -1$.

●Then, in order to calculate the optimal hedge ratio h_t according to the principle of minimum variance, we have

$$Min_{h_t} \quad Var_t\left(\frac{dS_t}{S_t dt} - h_t \frac{dF_{t,T}}{F_{t,T} dt}\right) \tag{4}$$

●Insert equation（2）and equation（3）to（4）:

$$Min_{h_t} \quad \sigma_{s,t}{}^2[1 + h_t^2(1+\delta_t^2 + 2\rho_{12,t}\delta_t) - 2h_t(1+\rho_{12,t}\delta_t)] \tag{5}$$

●Where $\delta_t = \sigma_{N,t}/\sigma_{s,t}$ Solve it for the result:

$$h_t^* = \frac{1+\rho_{12,t}\delta_t}{1+\delta_t^2 + 2\rho_{12,t}\delta_t} = \frac{1+\rho_{12,t}(\sigma_{N,t}/\sigma_{s,t})}{1+(\sigma_{N,t}/\sigma_{s,t})^2 + 2\rho_{12,t}(\sigma_{N,t}/\sigma_{s,t})} \tag{6}$$

In fact, According to equation（3）, the conditional variance of the yield of the stock index futures is $\sigma_{f,t}^2 = \sigma_{s,t}^2 + \sigma_{N,t}^2 + 2\sigma_{s,t}\sigma_{N,t}\rho_{12,t}$, the covariance between the yield of stock index futures and spot stock index is $\sigma_{sf,t} = \sigma_{s,t}^2 + \sigma_{s,t}\sigma_{N,t}\rho_{12,t}$ .we have

$$h_t^* = \frac{1+\rho_{12,t}(\sigma_{N,t}/\sigma_{s,t})}{1+(\sigma_{N,t}/\sigma_{s,t})^2 + 2\rho_{12,t}(\sigma_{N,t}/\sigma_{s,t})} = \frac{\sigma_{sf,t}}{\sigma_{f,t}^2} \tag{7}$$

2.4 Empirical model

As the conditional correlation coefficient between ε_{1t} and ε_{2t} is not a constant , D-BEKK-ECM-BGARCH model is needed for the calculation of optimal hedge ratio.

(1) Assume s is the log price of the spot stock index, f is the log price of the stock index futures, then the yield of them can be expressed respectively as:

$$r_{s,t} = s_t - s_{t-1}, \quad r_{f,t} = f_t - f_{t-1} \tag{8}$$

(2) The mean equation of D-BEKK-ECM-BGARCH model:

$$\begin{bmatrix} r_{s,t} \\ r_{f,t} \end{bmatrix} = \begin{bmatrix} C_s \\ C_f \end{bmatrix} + \begin{bmatrix} \lambda_s z_{t-1} \\ \lambda_f z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{s,t-1} \\ \varepsilon_{f,t-1} \end{bmatrix}$$
$$\varepsilon_t | \Omega_{t-1} \sim N(0, H_t) \tag{9}$$

Where z_{t-1} is the error correction term and $z_{t-1} = s_{t-1} - (\hat{\alpha} + \hat{\beta} f_{t-1})$.

(3) The variance equation of D-BEKK-ECM-BGARCH model:

$$vec(H_t) = vec(C'C) + vec(A'\varepsilon_{t-1}\varepsilon_{t-1}{}'A) + vec(B'H_{t-1}B)$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} \sigma_{s,t}^2 & \sigma_{sf,t} \\ \sigma_{sf,t} & \sigma_{f,t}^2 \end{bmatrix}. C = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix}、 A = \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix}、 B = \begin{bmatrix} \beta_{11} & 0 \\ 0 & \beta_{22} \end{bmatrix} \tag{10}$$

Where

$$vec(H_t) = \begin{bmatrix} h_{ss,t} \\ h_{ff,t} \\ h_{sf,t} \end{bmatrix} \qquad \begin{aligned} h_{ss,t} &= C_{11}^2 + \beta_{11}^2 \times h_{ss,t-1} + \alpha_{11}^2 \times \varepsilon_{s,t-1}^2 \\ h_{ff,t} &= C_{22}^2 + C_{12}^2 + \beta_{22}^2 \times h_{ff,t-1} + \alpha_{22}^2 \varepsilon_{f,t-1}^2 \\ h_{sf,t} &= C_{11} \times C_{22} + \beta_{22} \times \beta_{11} h_{sf,t-1} + \alpha_{11} \times \alpha_{22} \varepsilon_{s,t-1} \varepsilon_{f,t-1} \end{aligned}$$

We can get the optimal hedge ration

$$h_t = \frac{Cov(\varepsilon_{st}, \varepsilon_{ft})}{Var(\varepsilon_{ft})} = \frac{\sigma_{sf,t}}{\sigma_{f,t}^2} = \frac{h_{sf,t}}{h_{ff,t}} \qquad (11)$$

3 Date and Empirical Analysis

3.1 Data selection

Select the following 4 stocks: Gongshangyinhang, zhongguoyinhang, wankeA, shenfazhan which stand a big share by the funds. Set up portfolios in accordance with their market value. The prices of the stocks are the closing prices from 1st June 2007 to 30th April 2008. Download the corresponding data of Hushen300 spot and future from “Wenhhua futures terminals”, the data of spot goods are the closing prices, the data of the future stems from the closing prices of the futures with the coming month delivery month. Match up all the above-mentioned data in sequence of time. There are totally 209 groups of data.

3.2 Calculation of β risk factor

With the equation $r_{s,t} = s_t - s_{t-1}$, respectively calculate the yields of the above-mentioned 4 stocks as well as the Hushen300 spot goods index. And then calculate the yield of the stock portfolio r_p , regress with the yield of Hushen300 spot index r_m, and the coefficient of r_m is 0.38 which demonstrates that in comparison with Hushen300 index, the β coefficient of the portfolio is 0.38.

3.3 Calculation of hedge ratio h

Due to the complexity of D-BEKK-ECM-BGARCH model, this passage taking advantage of EVIEWS directly calculate the optimal hedging ratio h_t of Hushen300 future index to spot index. The optimal hedging ratios are depicted in the following Figure 1:

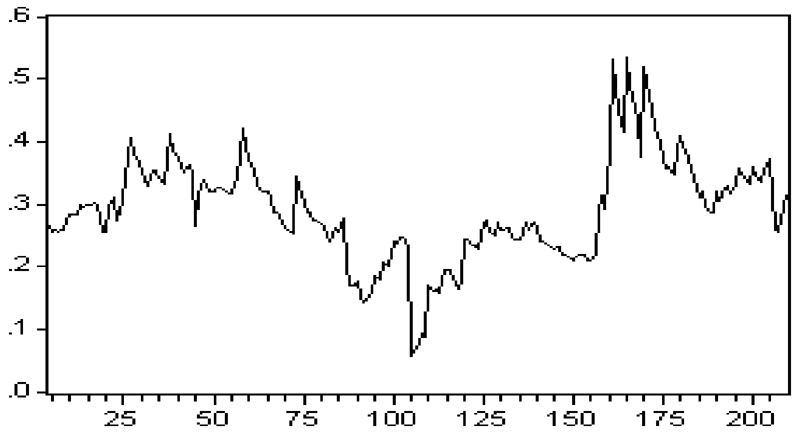

Figure 1 The Optimal Hedging Ratio

3.4 The efficiency of the hedge ratio

Analyze the effect of the hedging by Table 1

Table 1 The Effect of the Hedging

	Mean of the hedge ration	Variance of the portfolio	Reduction rate of variance
Unhedged portfolio	. 0	0.005628	0
Hedged portfolio	0.110503	0.000843	85.02%

From the figure we know that: after hedging the risk of the portfolio will be reduced significantly ; and the hedge ratio is less than 1 significantly in D-BEKK-ECM-BGARCH model, which means that the cost of the hedging is small.

4 Conclusions

According to the empirical Study of this article, HuShen 300 index futures contracts can disperse risk effectively. The optimal hedge ratio calculated by D-BEKK-ECM-BGARCH model can maximize the reduction of portfolio risk with a very small cost.

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